J Math Chem (2010) 47:1263–1273
DOI 10.1007/s10910-009-9650-1

ORIGINAL PAPER

Modeling a clinical incineration process using fuzzy autocatalytic set

Tahir Ahmad · Sabariah Baharun · Khairil Anuar Arshad

Received: 16 June 2009 / Accepted: 25 November 2009 / Published online: 16 December 2009

Abstract In this paper we have investigated and explored the realm of fuzzy graph in its relation to autocatalytic sets. A new concept namely fuzzy autocatalytic set have been defined. This relation has produced some results in the forms of theorems which have been proven. A clinical waste incineration process has been modeled using this new concept indicates to be more accurate than using crisp graph.

Keywords Fuzzy graph · Fuzzy autocatalytic set · Incineration process · Chemical engineering

1 Introduction

Ever since Zadeh [20] introduced the idea of fuzzy set theory by utilising the concept of grade membership, numerous researchers have been concerned with the properties and applications of fuzzy sets [4,8,9,11,17]. This is due to the fact that certain aspects of reality such as complexity and ill-defined situations always escape most crisp mathematical models. Thus the crisp models usually inadequate in describing the whole process of the systems.

Fuzzy graph was another extension of fuzzy theory's application in its relation to graph theory. Rosenfeld [14] has defined fuzzy graph in which he has considered fuzzy graph to consist both fuzzy set for vertices as well as for the edges given as follows:

Definition 1.1 Fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ $\forall x, y \in S$, we have $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

T. Ahmad (✉) · S. Baharun · K. A. Arshad
Fuzzy Research Group, Department of Mathematics & Ibnu Sina Institute for Fundamental Science Studies, Faculty of Science, Universiti Teknologi Malaysia (UTM), 81310 Skudai, Johor, Malaysia
e-mail: atieawang@gmail.com; tahir@ibnusina.utm.my

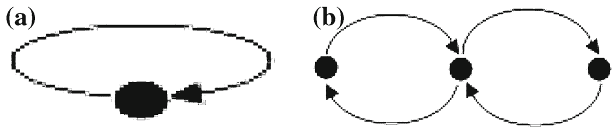

Fig. 1 **a** A 1-cycle, the simplest ACS. **b** An irreducible graph but not cycle

Yeh and Bang [19] also coined a special case of graph fuzziness where only the edges are fuzzy and the vertices remain as a crisp set. Blue et al. [1,2] further generalized the catalog of various types of fuzziness possible in graphs, which they called taxonomy of fuzzy graphs. These graphs can be classified into 5 types as follows:

Type 1: Fuzzy Set of Graphs
Type 2: Crisp Vertex Set and Fuzzy Edges
Type 3: Crisp Vertices and Edges with Fuzzy Connectivity
Type 4: Fuzzy Vertex Set and Crisp Edge Set
Type 5: Crisp Graph with Fuzzy Weight.

Even though these classifications cover the whole range of possible ‘fuzziness’ of a graph, however, a real working model of a system remains illusive. In the following sections we will present how these graphs are related to autocatalytic set proposed by Jain and Krishna [6] and how it help to model an incineration process.

2 Fuzzy autocatalytic set

The concept of autocatalytic set (ACS) was first introduced in the context of catalytically interacting molecules, [7,13]. However, Jain and Krishna [6] have formalized the autocatalytic set in terms of graph theoretical concept (see Fig. 1) as follows:

Definition 2.1 An autocatalytic set is a subgraph, each of whose nodes has at least one incoming link from a node belonging to the same subgraph (see Fig. 1).

The main idea for the definition of fuzzy autocatalytic set (FACS) is the merger of fuzzy graph concept as discussed in Sect. 1, in particular Type 3 and the concept of an autocatalytic set. The formal definition is given as follows:

Definition 2.2 Fuzzy Autocatalytic Set (FACS) is a subgraph each of whose nodes has at least one incoming link with membership value $\mu(e_i) \in (0, 1], \forall e_i \in E$.

The membership values for fuzzy edge connectivity for fuzzy graph are in the interval (0, 1]. These values constitute the entries of the adjacency matrix for fuzzy graph. With this difference in values to the adjacency matrix of fuzzy graph as compared to the value of 0 and 1 in the adjacency matrix of crisp graph would result in the difference of their algebraic structure. Such differences are presented in the following theorems.

Theorem 2.1 *If G is an FACS with adjacency matrix*

$$C_G = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \tag{2.1}$$

and for each $\alpha = 1, 2, \ldots, n$, *there exist* $\beta \neq \alpha$, $\beta = 1, 2, \ldots, \alpha - 1, \alpha + 1, \ldots, n$ *such that* $a_{\alpha\beta} \neq 0$, *then* $a_{ij}\ a_{si}\ a_{ts} \cdots a_{pl}\ a_{jp}$ *is a cycle.*

Proof Suppose G is an FACS with adjacency matrix C_G as above. Next consider the sequence of non-zero entries of C_G as $a_{ij}\ a_{si}\ a_{ts} \cdots a_{pl}\ a_{jp}$. Look

a_{ij} indicates there exist a link from j to i,
a_{si} indicates there exist a link from i to s,
a_{ts} indicates there exist a link from s to t,
$\vdots$
a_{pl} indicates there exist a link from l to p and
a_{jp} indicates there exist a link from p to j.

Since all links are connected and the path starts and ends at the same point (vertex), therefore the sequence in the form of
$a_{ij}\ a_{si}\ a_{ts} \ldots a_{pl}\ a_{jp}$ must be a cycle. □

FACS can be linked with Perron-Frobenius Theorem in particular its relation to the PFEs. This resulted in the following theorem.

Theorem 2.2 *If a graph is an FACS, then* $\lambda_1 > 0$.

Proof Suppose G_F is an FACS, therefore G_F must contain a cycle by Theorem 2.1. There exist a node that has at least one path to itself of length n with some fuzzy connectivity such that $\left(C^k\right)_{ii} > 0$ for $k \in Z^+$. We know that the summation of Eigen values is its trace; $\sum_{i=1}^{s} (C_{ii}) = \sum_{i=1}^{s} \lambda_i$, [10], similarly

$$\sum_{i=1}^{s} \left(C^k\right)_{ii} = \sum_{i=1}^{s} \left(\lambda_i^k\right). \tag{2.2}$$

Using the fact $\left(C^k\right)_{ii} > 0$ and Eq. 2.2, therefore

$$\sum_{i=1}^{s} (\lambda_i^k) > 0 \tag{2.3}$$

and from Eq. 2.3,

$$\left|\sum_{i=1}^{s} (\lambda_i^k)\right| > 0. \tag{2.4}$$

Now, using the triangle inequality, [18]

$$\sum_{i=1}^{s} \left|(\lambda_i^k)\right| \geq \left|\sum_{i=1}^{s} (\lambda_i^k)\right| \tag{2.5}$$

$$\text{Using Eq. 2.3, 2.4 and 2.5, therefore } \exists \ \lambda_i^k \ \text{ for some } \ i \left|\lambda_i^k\right| > 0. \tag{2.6}$$

Consequently, $\lambda_1 = \left|\lambda_i^k\right|$ is the dominant eigenvalue with the largest modulus greater than zero. □

3 An incineration process

Clinical or medical waste is defined as a waste which consists wholly or partly of human and animal tissue, blood or any other body fluids, excretions, drugs or pharmaceutical products, swabs or dressings, or syringes, needles, or sharp instruments that is hazardous to any person coming into contact with it. It also includes waste arising from medical, dental, veterinary, pharmaceutical or similar practice, investigation, treatment, care, teaching, or research, or collection of blood for transfusion that may cause infection to any person coming into contact with it.

Incineration is known to be the best available option for treating such waste particularly the pathological related waste. It appears to be the ideal solution to clinical waste management mainly because it has the capability of destroying hazardous components of a waste and of reducing a waste's volume by leaving only ash to dispose of. Although it is a preferred method of disposal, the air impurities associated with the burning of the waste is a major concern to the public [12]. This includes the product of incomplete combustion consisting of organic compounds which are toxic even at extremely low concentrations. Polychlorinated dibenzo-p-dioxin (referred to as dioxin) and polychlorinated dibenzo-furans (or furan) are two such toxic combustion products.

Since there is no fundamentally based mathematical model to explain the underlying principles involving the incineration process and operational control of the incinerator, a study was carried out to model the process using graph. A regional clinical waste incinerator facility in Malacca (schematic diagram given in Fig. 2), owned by Pantai Medivest Sdn Bhd known formerly as Tongkah Medivest Sdn Bhd was selected and modeled in the research [16]. It is estimated that 200 metric tons of clinical waste being generated throughout the southern region of Peninsular Malaysia every month from hospitals and clinics and this waste is being catered for by this facility.

The initial graph modeled in the study with the components of the incinerator denoted as vertices and the input-output variables as the edges [15] is seen to be static in nature (Fig. 3) and not much of its properties can be studied. Therefore, a dynamic model of the incineration process is sought for.

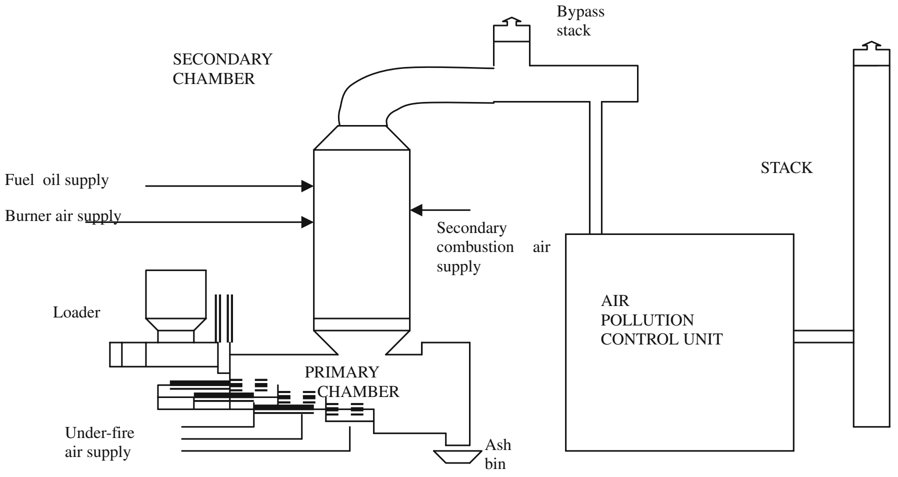

Fig. 2 The schematic diagram of a clinical waste incinerator

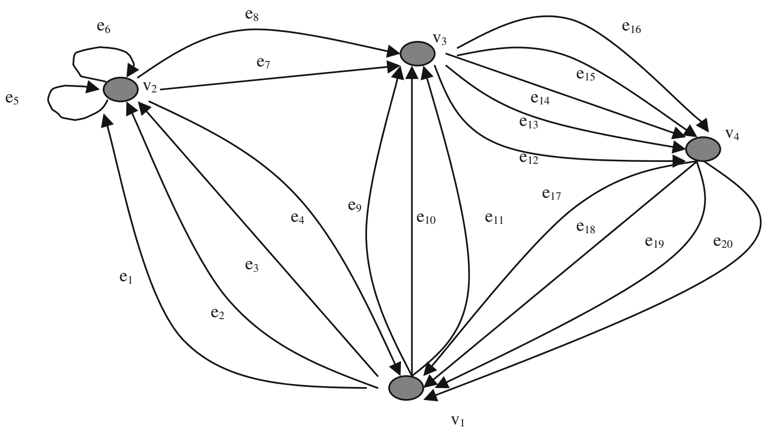

Fig. 3 Graph showing the association between input–output variables and different components of the incineration process

v_1: Environment
v_2: Primary combustion chamber (PCC)
v_3: Secondary combustion chamber (SCC)
v_4: Air pollution control unit (PCU)
e_1: Wood
e_2: Clinical waste
e_3: Underfire Air—Air 1
e_4: Ash
e_5: Temperature (Zone 1 and 2)—Temp1
e_6: Temperature (Zone 3)—Temp2
e_7: The product of combustion

e_8: Temperature at the inlet of the secondary chamber—Temp3
e_9: Fuel
e_{10}: Burner air supply—Air2
e_{11}: Secondary combustion air supply—Air3
e_{12}: Temperature at the outlet of the secondary chamber to the PCU—Temp4
e_{13}: Carbon monoxide
e_{14}: Carbon dioxide
e_{15}: Oxygen
e_{16}: Pollutants
e_{17}: Oxygen
e_{18}: Carbon monoxide
e_{19}: Carbon dioxide
e_{20}: Pollutants

4 A dynamic model

The four components of the incinerator plant were discarded (see Fig. 3) from graph and the emphasis of the study was to investigate the role played by the variables in the incineration process [16]. In this regard, the process was assumed to be taking place in one compartment; the incinerator. This led to the development of the dynamic graph G_d. $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the set of vertices represent six variables that play vital roles in the clinical waste incineration process, namely waste, fuel, oxygen, carbon dioxide, carbon monoxide and other gases including water, respectively. Fifteen edges of *E* represent the links of the variables in the process due to their catalytic relationship [16] given as follows.

(v_1, v_2) : *Waste particle interact with fuel in the chamber to initiate and aggravate the combustion process. Apparently, besides diesel, waste is also considered as part of fuel for the process.*

(v_1, v_3) : *One of the products of combustion based on chemical composition of waste specified by Green* [5] *is* O_2. *This means that waste is contributing a certain amount of* O_2 *to the chamber. In fact, the initial amount of* O_2 *in the chamber taken as the concentration dynamics of this variable relates to this interaction.*

(v_1, v_4) : CO_2 *is the product of combustion waste. Consequently, waste catalyzed the production of* CO_2.

(v_1, v_5) : CO *is also the product of combustion of waste. This gas is produced as a result of an insufficient amount of* O_2 *in the chamber.*

(v_1, v_6) : H_2O *and Pollutants are the products of combustion of waste. Therefore, waste catalyzed the production of these variables.*

(v_2, v_4) : *In the waste and fuels normally encountered, the major constituents include carbon and hydrogen. The fuel (diesel) used in the chamber must also contribute to the production of* CO_2.

(v_2, v_5) : *In the case where* O_2 *is insufficient, there is trace of production of* CO *in the chamber. Bruner* [3] *explained the production of* CO *by the following equation.*

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O + CO\,(\text{trace}) \tag{4.1}$$

(v_2, v_5) : *Equation (4.1) indicates that water is catalyzed by fuel.*
(v_3, v_1) : *11% of waste constitutes of* O_2 [5]. O_2 *is part of the waste especially at its initial stage in the chamber.*
(v_3, v_4) : O_2 *catalyzes the formation of* CO_2.
(v_3, v_5) : *The equation*

$$C + \frac{1}{2}O_2 \rightarrow CO \tag{4.2}$$

shows that O_2 *is the variable that motivates the formation of* CO. CO *is produced where insufficient oxygen is provided to completely combust a fuel.*
(v_3, v_6) : O_2 *and other molecules presence in waste and also fuel such as in Eq. 4.1.*
(v_4, v_6) : CO_2 *is a stable gas. But there are circumstances whereby, it will react with water vapor in the chamber to produce carbonic acid which we classified as* v_6.
(v_5, v_4) : CO *can react with* O_2 *to form* CO_2. *The reaction took place when excess amount of* O_2 *is in the chamber. It is the intermediary reaction between carbon and oxygen to form* CO_2 *and if* CO *was formed instead, it will automatically react or mix with more oxygen to form* CO_2.
(v_6, v_1) : *About 23% of waste constitutes of water* (H_2O) *as shown by Green* [5].

By the definition of autocatalytic set given in Sect. 2.1 and catalytic relationship described earlier, we can have the following theorem by construction.

Theorem 4.1 *The graph* $G_d(V, E)$ *is an autocatalytic set.*

5 A fuzzy graph type 3

Eventhough Rosenfeld [14] and shortly Yeh and Band [19] have given the definitions of a fuzzy graph, however, we can formalized five types of fuzziness possible in graphs as described by Blue et al. [1,2] in the following.

Definition 5.1 Fuzzy graph is a graph G_F satisfying one of the fuzziness (G_F^i of the i^{th} type) or any of its combination:

i. $G_F^1 = \{G_{1_F}, G_{2_F}, G_{3_F}, ..., G_{n_F}\}$ where fuzziness is on G_{i_F} for $i = 1, 2, 3, \ldots, n$.
ii. $G_F^2 = \{V, E_F\}$ where the edge set is fuzzy.
iii. $G_F^3 = \{V, E\,(t_F, h_F)\}$ where both the vertex and edge sets are crisp, but the edges have fuzzy heads and tails.
iv. $G_F^4 = \{V_F, E\}$ where the vertex set is fuzzy.
v. $G_F^5 = \{V, E\,(w_F)\}$ where both the vertex and crisp sets are crisp but the edges have fuzzy weights.

The fact that crisp graph is a special case of a fuzzy graph can be stated in the following theorem.

Theorem 5.1 *Every crisp graph is a fuzzy graph.*

Proof Suppose $G(V, E)$ is a crisp graph. It can be considered as $G = (\sigma, \mu)$ which is a pair of function $\sigma : V \rightarrow \{0, 1\}$ and $\mu : V \times V \rightarrow \{0, 1\}$. It immediately fulfills the definition of fuzzy graph given by Rosenfeld [14] as well as definition given by Yeh and Bang [19]. □

In view of Definitions 2.1 and 2.2 in Sect. 2, Theorem 5.1 leads to the following corollaries.

Corollary 5.1 *Every Autocatalytic Set is a fuzzy graph.*

Corollary 5.2 *Every Fuzzy Autocatalytic Set is also a fuzzy graph.*

The definition of fuzzy graph given in Definition 5.1 was designed to formulate a fuzzy graph for the clinical waste incineration process (see Fig. 2). The graph also happens to be FACS. In order to do that, we need to introduce several definitions.

Definition 5.2 Let $e_i \in E$. The fuzzy head of e_i denotes as $h(e_i)$ and the fuzzy tail $t(e_i)$ are functions of e_i such that $h : E \rightarrow [0, 1]$ and $t : E \rightarrow [0, 1]$ for $e_i \in E$. A fuzzy edge connectivity is a tuple $(t(e_i), h(e_e))$ and the set of all fuzzy edge connectivity is denoted as $C = \{(t(e_i), h(e_e)) : e_i \in E\}$.

In relation to our system, $t(e_i) = 1, \forall e_i$ since each variable was taken as a whole before it evolved to other variables. The membership value of fuzzy head (citealtSa05 is based on the reaction taken place and the strength of connection to the other variables in our system. By taking the membership value for each fuzzy edge connectivity as

$$\mu(e_i) = \min\{t(e_i), h(e_i)\} \tag{5.1}$$

in Theorem 4.1 (see Fig. 4), we can state the following theorem.

Theorem 5.2 *The graph for the clinical waste incineration process can be represented as* $G_{d_F} = (V, E(\mu(e_i)))$ *for* $i = 1, 2, 3, \ldots, 15$ *where* $\mu(e_i) = \min\{t(e_i), h(e_i)\}$.

By Definitions 2.2 and 5.1, our fuzzy graph for the clinical incineration is an FACS as well as a fuzzy graph of Type 3 (see Fig. 5).

6 Implementation

The same color and thickness of each link in the crisp graph G_d (see Fig. 4) reveals that the connectivity between the vertices in the graph is the same. However, as compared to fuzzy graph G_{d_F} in Fig. 6, the greater the value of connectivity between the vertices, the thicker is the link between them. The different color signifies the different range of

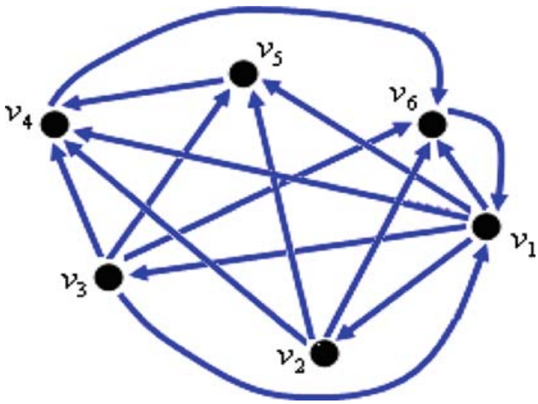

Fig. 4 Crisp graph $G_d(V, E)$ of the incineration process

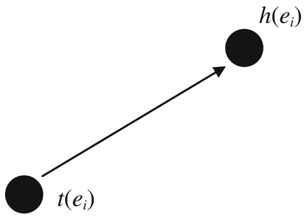

Fig. 5 Fuzzy head and tail

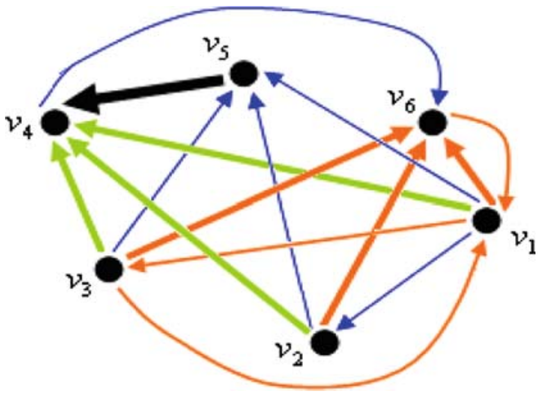

Fig. 6 Fuzzy graph of type 3 G_{d_F} for the clinical incineration process

membership value for the fuzzy edge connectivity. Thus, a difference in the result of the model would be expected since each model depends on the adjacency matrix of its graph. The graph dynamic procedure discussed, Sabariah [16] is applied to each graph. The results of concentration dynamics of the initial phase for both models are given in the following Table 1.

The rate of change of Fuel is negative for fuzzy graph as compared to the positive value for the crisp graph. The concentration value of Fuel should be decreasing as it was being consumed during the incineration process. Furthermore, the value obtained for CO by using fuzzy graph showed the formation of the gas is minimal in the incineration process. This is due to the assumption of perfect burning for the process.

Table 1 Comparison between models using crisp and fuzzy graph

x_i	Variable	Rate of change using crisp graph	Rate of change using fuzzy graph
1	Waste	−0.45300	−0.20671
2	Fuel	0.13600	−0.02952
3	O_2	−2.49600	−0.57228
4	CO_2	0.94500	0.56995
5	CO	0.94500	0.00002
6	H_2O & other pollutants	0.70900	0.23854

Table 2 Comparison between depletion sequence using crisp and fuzzy graph

	Crisp graph	Fuzzy graph
Sequence of depleted variables	Fuel, O_2, CO, CO_2	Fuel, CO, O_2, waste
Products of process signified by the variables left at the end of procedure	Waste and H_2O & other pollutants	CO_2 and H_2O & other pollutants

Moreover, the results of graph dynamics for both models can be compared by looking at the sequence of depleted variables and also the set of the variable left at the end of the procedure that signify the products of the incineration process as shown in Table 2.

The sequence of the variables removed from the system during the incineration process is more accurate for fuzzy graph as compared to the crisp graph. Fuzzy graph also indicated the products of the process as CO_2 and H_2O which confirmed to the real process as noted by Bruner [3].

7 Conclusion

Fuzzy Autocatalytic Set (FACS) has been defined and implemented in the modeling of the incineration process. The development of the theory has also been outlined in the paper along with the comparisons made to the results on both applications of crisp and fuzzy graph in the modeling. The results shown using this new concept prove to be more accurate than using crisp graph.

Acknowledgments This study is part of a research funded by the Research Management Centre, Universiti Teknologi Malaysia, with vote number 71904.

References

1. M. Blue, B. Bush, J. Puckett, Applications of Fuzzy Logic to Graph Theory. Los Alamost National Laboratory. LA-UR-96-4792 (1997)

2. M. Blue, B. Bush, J. Puckett, Unified approach to fuzzy graph problems. Fuzzy Sets Syst. **125**, 355–368 (2002)
3. C.R. Bruner, *Handbook of Incineration Systems* (McGraw Hill Inc., New York, USA, 1991)
4. I. Gitman, M.D. Levine, An Algorithm for Detecting Unimodal Fuzzy Sets and Its Application as a Clustering Technique. IEEE Trans. Comput. **19**(7), 583–593 (1970)
5. A.S. Green, *Medical Waste Incineration and Pollution Prevention* (Van Nostrand Reinhold, New York, 1992)
6. S. Jain, S. Krishna, Autocatalytic sets and the growth of complexity in an Evolutionary Model. Phys. Rev. Lett. **81**, 5684–5687 (1998)
7. S.A. Kauffman, Cellular homeostasis, epigenesist and replication in randomly aggregated macromolecular systems. J. Cybern. **1**, 71–96 (1971)
8. A. Kandel, L. Yelowitz, Fuzzy chains. IEEE Trans. Syst. Man. Cybern. **4**(5), 472–475 (1974)
9. J.M. Keller, H. Tahani, Implementation of conjunctive and disjunctive fuzzy logic rules with neural networks.. Int. J. Approx. Reason. **6**, 221–240 (1992)
10. B. Noble, J.W. Daniel, *Applied Linear Algebra*, 2nd edn. (Prentice Hall Inc., Englewood Cliffs, New Jersey, 1977)
11. A. Pathak, S.K. Pal, Fuzzy grammars in syntactic recognition of skeletal maturity from x-rays. IEEE Trans. Syst. Man. Cybern. **4**(5), 657–667 (1986)
12. P.A. Reinhardt, J.G. Gordon, *Infectious and Medical Waste Management* (Lewis Publishers Inc., Michigan, 1991)
13. O.E. Rossler, A system theoretic model of biogenesis. Z. Naturforsch. **26**, 741–746 (1971)
14. A. Rosenfeld, in *Fuzzy Graphs*, ed. by L.A. Zadeh, K.S. Fu, M. Shimura Fuzzy Sets and Their Applications to Cognitive and Decision Processes. (Academic Press, New York, 1975)
15. B. Sabariah, A. Tahir, A.A. Khairil, M. Rashid, I. Zamri, Graphical presentation of a clinical waste incineration process, in *Proceedings of the 10th national symposium of mathematical sciences*. Johor Bahru, Johor, (2002), pp. 109–115
16. B. Sabariah, Modeling of a clinical waste incineration process using novel fuzzy autocatalytic set: a manifestation of mathematical thinking. Thesis PhD, UTM (2005)
17. S. Tamura, S. Higuchi, K. Tanaka, Pattern classification based on fuzzy relations. IEEE Trans. Syst. Man. Cybern. **1**(1), 61–66 (1971)
18. D. Varberg, E.D. Purcell, *Calculus*, 7th edn. (Prentice Hall, New Jersey, 1997)
19. R.T. Yeh, S.Y. Bang, in *Fuzzy Relations, Fuzzy Graphs and Their Applications to Clustering Analysis*, ed. by L.A. Zadeh, K.S. Fu, M. Shimura Fuzzy Sets and Their Applications to Cognitive and Decision Processes. (Academic Press, New York, 1975)
20. L.A. Zadeh, Fuzzy sets. Inf. Control **8**, 338–359 (1965)